

potentially puzzling set of numbers (fractions) and a challenging mathematical concept (percents).

Math-to-World Connections

By connecting new concepts to what they know about the world, students deepen their understanding of mathematics. For example, third graders studying measurement benefit from relating real-world containers to abstract units of measurement. Though you can tell students how many ounces are in a cup, how many cups are in a pint, how many pints in a quart, and how many quarts in a gallon, they are more likely to understand these relationships (and remember them) by comparing a standard school-issue carton of milk (1 cup) with pint, quart, half-gallon, and gallon containers of milk. By looking at, holding, and pouring with these different everyday containers, students make tactile and visual connections between familiar real-world objects and abstract information. While it will be necessary for them to learn the name of each container's volume, making this math-to-world connection gives students a concrete reference point that they can recall at any time from memory.

Questioning in Mathematics

Questions are at the heart of learning. Our need to question is what drives human beings to new, deeper, more accurate understandings as well as new fields of study; before the age of quantum physics, no one knew to ask, "What is dark matter?"

Students benefit from direct and explicit instruction in how to use questioning as a math comprehension strategy. However, explicit teaching of questioning as a math comprehension strategy is not sufficient to help students become questioners by habit. The asking of good questions must be a part of the culture of the classroom. Teachers often get into a questioning rut, posing one-answer, read-my-mind sorts of questions: "What is a square number?" "How many sides does a rhombus have?" While there will always be a place for these kinds of questions, they do ask very little of a learner, sometimes requiring no thinking beyond recall.

In *Good Questions for Math Teaching: Why Ask Them and What to Ask*, Lainie Schuster and Nancy Canavan Anderson describe good

questions as "open-ended, whether in answer or approach" (2005, 3). In a classroom where questioning is valued, you should model types of questions that push learners beyond one answer in their mathematical thinking: "Is there a pattern here?" "Will this always work?" "Why does this work?" These are the kinds of questions that encourage learners to explore new territory.

Another way for teachers to learn how to ask good questions is to think about how we listen. Early childhood teacher Ann Carlyle said, "I try never to ask children a question that I know the answer to." As she described it, asking such questions limited her listening. When you ask, "What is seven plus seven?" you are listening for fourteen. In contrast, when you ask, "How did you solve seven plus seven?" you have no idea what a child is going to say, and hence you must listen more closely.

By modeling good questioning, we teach children the kinds of thinking we value. When we respond to students who ask, "Is this right?" with our own question, "What do you think?" we are communicating our belief that the quality of their thinking is more important than the answer alone.

While students benefit from being taught a general questioning habit of mind, there are types of questions that specifically help students understand math. These question types fall into four categories:

Four Types of Math Questions

questions that help students get started

questions that help students get unstuck

questions that help students check work

questions that help students go deeper

Getting Started

Questioning moves readers forward. When readers pick up a book, they pose a number of questions to themselves: "What will this be about? Will I like it as much as the author's other books? Will it be too hard or too easy for me?" As they delve into the first pages, new questions move them along: "Where is this taking place? Who is the main character? Is this happening now, or long ago?" In finding answers to

these initial, stage-setting questions, readers fully immerse themselves in the worlds of their books.

Questions serve a similar purpose in mathematics. Faced with a new problem, activity, or game, young learners often find themselves at a loss. Whether they are beginning work on an investigation of number patterns, using geoboards to explore common fractions, or solving a story problem, despite the clearest of instructions, students often do not know where to start.

When students come to me with their getting-started questions ("What do I do?"), I respond with a compliment ("I'm glad you asked!") and then pose questions of my own:

- ◆ "Do you remember what I said? If not, is there someone at your table you can ask? Is there somewhere you can look?" I want to foster responsibility and independence.
- ◆ "Do you have the materials you need? Look around—what are other kids using?" Sometimes students can be quite passive in the classroom. Again, I want to encourage their active responsibility in their own work.
- ◆ "What's the very first thing you are going to do? And then what?" If the child can answer the first question, the answer to the second is often "Oh! I get it." If not, I know I have reteaching to do.

Eventually I return the questions to the students: "Have you asked yourself . . . what I told you to do today? what materials you need? how you plan to get started?"

Learning how to ask themselves getting-started questions is particularly helpful for students who approach math with trepidation. These learners are often caught staring blankly at their work. They need explicit instruction on how to crack open a math problem so they can make use of what they do know. Questioning themselves and others helps them get started.

Getting Unstuck

Questions help readers navigate confusing parts of their books. When understanding breaks down, it is critical for readers to notice their

confusion and to seek clarification: "Wait a minute—what happened to the dog?" "Why did they leave in the middle of the show?" "Whoa—what does *spontaneously combusted* mean?" Readers learn to respond to such questions in a variety of ways: turning to a more experienced reader for definitions, rereading, examining illustrations, or looking at another text for additional information.

Getting-unstuck questions help students deal with the inevitable confusion they will experience while exploring mathematics. At some point a strategy will dead-end, or a pattern will start acting unpredictably. These are challenging moments; confusion is an uncomfortable sensation, and students often feel helpless in the grips of it. Though you will need to teach students how to pose getting-unstuck questions, doing so will help diagnose their roadblocks and point to a way to get back on track: "Where did my pattern stop working?" "If I can't find any more fourths this way, what other strategies can I try?" "Did I read the question right?" "Did I make any mistakes so far?" Such questions prompt learners to reread their work, looking for where math went awry or where confusion set in, seeking help from another person, or turning to another resource (a calculator, for example).

Checking Work

Students can make good use of questioning to evaluate the reasonableness or accuracy of their work. Although learners typically turn to the teacher or another authority—such as the kid known to be good in math—to find out if they are right, the habit of questioning helps learners both verify their solutions on their own and deepen their understanding.

When students ask me, "Is this right?" my answer is "What do you think?" After all, the answer will be right on its own merits, not because I deem it right. If the math is not too hard, students should be able to determine if their solutions are accurate. They can do this by asking themselves a series of questions.

One of the most valuable questions students can ask is "Does my answer make sense?" This is a giant step away from "Is my answer right?" because it encourages students to look at the answer in context. Often students rely on a dependable procedure to determine the correctness of an answer—"the answer must be right because I

subtracted the right way”—without considering the reasonableness of the solution. When we teach students to evaluate the reasonableness of their work (by considering the magnitude of the number and the context, for example), they can spot significant errors quickly (and are reminded again that we value reasonableness!).

Next, I want students to ask themselves, “How do I know it is right?” They may verify their work by checking it over, using a testing technique (e.g., adding to test the results of subtraction), or using an alternative method to solve the problem again, including such technology as a calculator. This is really valuable work. Students become both mathematically powerful and confident when they depend upon themselves to prove their work, instead of relying on the teacher as the arbiter of correctness.

Going Deeper

The power of questioning rests in its potential for leading learners into new territory. Neither reading nor the study of mathematics is a finite activity. When we are reading wide-awake, we are able to go well beyond the literal meaning of the text to challenge our own ideas, create new theories about the world, and gain deeply personal insights. We are often impelled to do this by raising and persistently pursuing questions: Why are humans so prone to do evil? What does it mean to be a friend? Where does a person draw the line between self-interest and the needs of the community? These are the kinds of questions that have no answer but drive us to learn and think all the same.

Elementary mathematics is the gateway to a limitless universe of questions. As much as we value a solid skill base and sound conceptual knowledge, it is at least as vital to help students develop open and questioning minds. We want students to be skilled problem solvers, and to that end they must learn to be astute questioners.

When my first class of fourth and fifth graders was engaged in a study of factors, they became curious about the number zero. They were discovering all sorts of interesting things about patterns in multiples, square numbers, primes, and even and odd numbers. Zero began to perplex them: What was it, anyway? Even or odd? This deeply provocative question (well, it was deeply provocative to them) took us on a bit of a detour from our focus on multiplication, but

what a payoff! Together, we looked at zero through the lens of patterns, number theory, the history of mathematics, and contemporary understanding. It was a richly engaging mathematics adventure with lessons that ultimately went beyond understanding the nature of zero.

Inferring and Visualizing in Mathematics

In books as in life, meaning is often found beyond the immediate details. Visualizing (including other sensory images) is a way to enhance our ability to infer and monitor our understanding of meaning. When we succeed in building sensory images while reading Laura Ingalls Wilder's *The Long Winter* (1940), for example, we feel the horror of the bitter, cold, endless howling winds that the Ingalls family endures for seven dark months. We admire Laura's strength of character and meet the arrival of spring with relief and gratitude. Visualizing helps us monitor understanding, too. When we expect to make mental images and then have trouble building them, it's difficult for us to feel any connections. In such cases we've probably missed an important detail or two, or the book may be too hard—or maybe even badly written!

Visualizing in mathematics serves these same functions and more. Proficient math learners expect to have sensory connections to the math they are doing. Whether it is seeing a mental number line or grid, visualizing mental manipulation of numerals or shapes, or seeing story problems in context, sensory images deepen appreciation and comprehension *and* signal problems. Sensory images that are incomplete or don't make sense drive learners back to repair meaning.

Visualizing is also a math comprehension strategy that helps learners solve problems and deepen understanding. Because it is so important, teachers incorporate it into lesson design. However, even though we may talk explicitly about visualizing as a math comprehension strategy and offer guided practice to students (“What do you picture when I say triangle?” or “Try imagining the story problem; will Mary have more dinosaurs after Stewart leaves or fewer?”), students still need support on *how* to create appropriate images that they can recall later from memory (in many cases, learners do not yet have visual or sensory images for the material they are learning). By incorporating manipulative materials, drawings, and everyday objects